Physical understanding of an echo-Doppler test with voice-induced vibration

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The physical understanding of a method of detecting mammalian cancer via vocalization during a normal echo-Doppler test is provided. The backscattered ultrasound frequency in the case of a vocal humming resonating in the chest wall is computed: the overall effect is that the signal/noise ratio could be easily improved at no cost. Clinical results are to appear separately elsewhere.

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I. INTRODUCTION

Doppler ultrasonography is a non invasive, non-x ray diagnostic procedure that can improve the accuracy of clinically and mammographically detected abnormalities. It is widely used to differentiate the different tissue components in breast and as a problem-solving tool in the radiologist's armamentarium. A transducer (the probe) sends a series of short ultrasound pulses into the tissue and periodically pauses to listen for the returning sounds: via the Doppler effect we can detect the direction, velocity and turbulence of blood flow in a vascularization. The idea of setting the target into forced vibration has also been exploited in many different techniques (a general review of several is given in Ref.[1]), for instance the mechanical properties of a tissue under forced oscillation can be revealed. Occasionally ultrasounds themselves (at low frequency) are used to force the motion of our target.[2]

The method we examine here is based on a Doppler ultrasound measurement under a vocal-forced oscillation, with application to breast scan. If the patient is asked to hum with voice, *i. e.* vocalize, during a normal echo-Doppler mammalian test, the sound vibrations may set into motion the different connective tissues inside breast. The Doppler ultrasound probe, which can detect movement, may thus be able to provide a signal even stronger than the one due to the natural blood flow.

In this paper we calculate the Doppler ultrasound signal reflected back by a little (~ 5 mm) spherical inclusion, when the latter is entrained by the vibrations of a viscoelastic medium oscillating at voice frequency. With a comparison with the typical Doppler signal of the usual non-vibrating situation we show that a good signal/noise ratio could be easily obtained.

II. EVALUATING MINOR CONTRIBUTIONS

Let us consider a spherical inclusion (a lesion) in a viscoelastic medium (the mammalian tissues). A sound wave propagating in an elastic medium can of course carry on in translational motion any object embedded in the medium. In the same time other effects may appear, for instance the periodic pressure variations of sound can make the object pulsate (shrink and swell) with the same frequency. In the specific case of a Doppler ultrasound measurement in a tissue already vibrating at the frequency of the human voice, another issue may *a priori* affect the measurement. The velocity of propagation of ultrasounds in a moving (vibrating) tissue is indeed different from that in a motionless medium. In principle ultrasounds scattered back by the spherical inclusion (the target) and collected back by the probe may have a significantly different frequency in either situation.

Let us consider the effect of having a moving medium in the unidimensional case, for sake of clearness. First suppose that both the medium and the probe are at rest and blood in the lesion's capillarization, by which the ultrasound is scattered back, is moving with velocity u (e. g. toward the probe) with respect to the motionless medium. If the emitted frequency is ν, the collected back frequency ν' is

$$\nu' = \nu \frac{1 + u/c}{1 - u/c} \sim \nu \left(1 + \frac{2u}{c}\right) \tag{1}$$

where $c \sim 1540$m/s is the sound speed in the human body.

Now suppose that a sound wave is propagated in the medium, which now is vibrating with velocity v: we can fairly assume that during the period in which the ultrasound travels from and back to the probe v is constant.

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Actually for an object 5 cm far, the ultrasound traveling time is $\sim \frac{10cm}{c}$, about 1/100 of the period of a normal 200 Hz sound wave. We also suppose that the probe, held by the operator, is still at rest and that blood in our target is moving again with the same velocity u toward the probe, just as if our target would not have been entrained by the motion of the surrounding medium. We have then in sequence: a source at rest (the probe), the medium, moving with velocity v toward the source, and the target, which is moving with velocity u with respect to the probe. The collected back frequency ν' is approximately

$$\nu' \sim \nu \frac{\left(1+\frac{u}{c+v}\right)}{\left(1-\frac{u}{c-v}\right)} \sim \nu \left(1 + (1+\frac{v^2}{c^2})\frac{2u}{c}\right) \qquad (2)$$

We have thus shown that corrections due to the simple motion of the surrounding tissues are negligible (second order in v/c). In other words, if the object is not entrained in translational motion by the wave, the signals observed by the echo-Doppler device with (2) and without (1) the medium vibration are nearly the same.[3] For the same reason, if the object is entrained by the medium and moves with velocity u for whatever reason, the signal received by the Doppler apparatus is always eq. (1) *no matter what the underlying tissue velocity is.* Voice has then no means of influencing the Doppler measurement *directly*, with a modification of the ultrasound propagation speed as described above or with a direct interaction with the device (whose operating frequencies are 1000 times higher). The only effects we need to take care of are the indirect ones: *the voice sets the inclusion into motion in some way and the probe detects the velocity u of the latter* no matter how the former is scattered or propagated.

Hence let us focus on the possible causes of motion for the inclusion and for each one let us calculate the respective velocity contribution u: the relative signal ν' follows through (1).

First of all we will give a rough estimate of the pulsation of the spherical lesion due to the periodic change in pressure[4] of the surrounding medium with the sound wave. There are three important consequences of the small dimensions of the lesion ($\sim$ 1cm) w.r.t. the voice wavelength:

- The external pressure p_{ext} of the surrounding medium is the same for the whole inclusion;
- The vocal sound wave may be regarded as plane;
- The natural frequencies of resonance of the inclusion are much higher than the frequency of the sound wave. This means that in the equation of motion for the pulsating lesion the inertia term is negligible w.r.t. the elastic term: as a consequence, internal and external pressures are perfectly balanced ($p_{ext} = p_{int} := p$) at any moment.

Friction can be neglected in this analysis of the pulsation motion because the absence of friction leads to the worst situation where the pulsation velocity is the greatest.

We can thus write

$$\frac{\partial p_{int}}{\partial \rho_{int}} \frac{d\rho_{int}}{dt} = \frac{dp_{ext}}{dt} = \frac{dp}{dt} \qquad (3)$$

$$c_{int}^2 \frac{d\rho_{int}}{dt} = -c_{int}^2 \frac{\rho_{int}}{V}\frac{dV}{dt} = i\omega p$$

$$-\frac{3\rho_{int}}{R} c_{int}^2 \frac{dR}{dt} = i\omega p$$

where c_{int} is the sound velocity inside the pulsating object and ρ_{int}, R, V its density, radius and volume respectively. ρ is the density of the surrounding medium and ω the sound wave pulsation instead (a $e^{i\omega t}$ time dependence is always understood, here and in the rest of the paper). For a plane sound wave we have $p = \rho v c$, where v is the velocity perturbation propagating at speed c, while $\frac{dR}{dt} = u_{puls}$ is the pulsation velocity of the lesion's interface, the one seen by the Doppler probe. For the pulsation velocity we thus obtain (discarding the i phase factor)

$$u_{puls} = \frac{\omega \rho v c R}{3\rho_{int} c_{int}^2} \qquad (4)$$

Let us compare the latter quantity with the result of a pure translational motion. Suppose for the moment that the lesion is carried with a velocity u_{transl} which is not very different form the bulk medium velocity v: this working hypothesis is reasonable since the inclusion is not very different from the surrounding tissues. In such a situation the medium "does not sense" the presence of the inclusion and transfers to the object a moment per unit time which is the very same necessary for moving an equal volume of medium, that is $\rho V \frac{dv}{dt}$. We obtain the "dynamical Archimedes' law"

$$\rho_{int} V \frac{du_{transl}}{dt} = \rho V \frac{dv}{dt} \qquad (5)$$

which allows us to write for a plane wave

$$u_{transl} = \frac{\rho}{\rho_{int}} v \qquad (6)$$

From this we compute the ratio

$$\frac{u_{puls}}{u_{transl}} = \frac{\omega c R}{3 c_{int}^2} \sim \frac{1}{100} \qquad (7)$$

for ω coincident with the typical human voice frequency.

From this we learn that the inclusion may be considered as a rigid object, as the interface velocity contribution due to its pulsation movement is much smaller than the translational velocity of the sphere. *The ultrasound frequency scattered back to the Doppler is almost totally due to the contribution of the translational movement of the object carried on by the sound wave.*

Thus we can finally proceed to give the exact formula as if the oscillating object was a rigid sphere (i.e. not pulsing).

III. THE MAIN TRANSLATIONAL CONTRIBUTION

Our starting point is the relation[6,7] which links the force F of a rigid sphere oscillating with pulsation ω in a medium:

$$F = \frac{4}{3}\pi R^3 \rho\omega^2 \left[A_1 \frac{h_1(kR)}{kR} - 6B_1 \frac{h_1(hR)}{hR} \right] \qquad (8)$$

where h_n's are the spherical Hankel function of the first kind, ρ is the density of the surrounding tissue (not that of the oscillating object!) and k and h are the longitudinal and transverse wavenumbers $k = \omega/c_L$, $h = \omega/c_T$ with $c_L = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_T = \sqrt{\mu/\rho}$ (see table I for typical values of μ and λ).

TABLE I. Typical values[7] for $\mu = \mu_1 + i\omega\mu_2$ and $\lambda = \lambda_1 + i\omega\lambda_2$

ρ	1060 kg/m^3
μ_1	2500 N/m^2
μ_2	15 N s / m^2
λ_1	$2.6 \cdot 10^9$ N/m^2
λ_2	~ 0 (at acoustic frequencies)

The (8) is obtained[6,7] from the potential theory, by writing the general solution on a basis of spherical Hankel functions. A general form of Hooke's law is the constraint which relates the pressure with the displacement of the lesion, so that both elasticity and viscosity are taken into account and moreover a general slip condition at the interface is provided[6]. The coefficients A_1 and B_1 in case of no slipping interface are[7]

$$A_1 = -\frac{(3 - 3ihR - h^2R^2)k^3R^3e^{-ikR}}{k^2R^2(1 - ihR) + (2 - 2ikR - k^2R^2)h^2R^2} u_0$$

$$B_1 = \frac{(3 - 3ikR - k^2R^2)h^3R^3e^{-ihR}}{3k^2R^2(1 - iha) + (2 - 2ikR - k^2R^2)h^2R^2} u_0 \qquad (9)$$

where u_0 is the maximum amplitude of displacement of the sphere. What really concerns us is the limit $kR \to 0$, as the sound wavelength is much longer than the radius R of the lesion. In this limit it is easy to show that (8) reduces to

$$F = 6\pi\mu R u_0 \left(1 - ihR - \frac{1}{9}h^2R^2\right) \qquad (10)$$

which is the exact expression for the problem[9] of an oscillating sphere *in a incompressible medium* (indeed the medium can be regarded as incompressible if the wavelength involved is large). In the expression above we can recover the translational velocity, as $\mu u_0 \sim i\mu_2\omega u_0 = \mu_2 u_{transl}$.

This is not the whole story yet, as F is the force exerted on a fluid with no other external forces by an oscillating object with a given law of motion. The case of an object set in motion by a moving (incompressible) fluid is different: first of all we need[10] to replace u_{transl} with the relative velocity $u_{transl} - v$, where v is the fluid velocity very far from the lesion, or equivalently, the velocity the whole fluid would have if there were no embedded objects in it, and then we have to add the "Archimedes' contribution" $\rho V \frac{dv}{dt}$ we discussed above. The final expression for the force acting on the spherical lesion is then

$$F = -6\pi\mu_2 R(u_{transl} - v)\left(1 - ihR - \frac{1}{9}h^2R^2\right) + \rho V \frac{dv}{dt} \qquad (11)$$

By Newton's law for the oscillating lesion $F = \frac{4}{3}\pi R^3 \rho_{int} \frac{du_{transl}}{dt}$ and substituting

$$u_{transl} = \frac{\frac{9\mu_2}{2R^2\omega}\left(1 - ihR - \frac{1}{9}h^2R^2\right) + i\rho}{\frac{9\mu_2}{2R^2\omega}\left(1 - ihR - \frac{1}{9}h^2R^2\right) + i\rho_{int}} v \qquad (12)$$

The above formula can be also obtained as the $k \to 0$, $\lambda + 2\mu \to \infty$ limit of equation (44) in Ref.[5], with the substitution $F_p \to \frac{4}{3}\pi R^3 \rho_{int} iwu_{transl}$ and $u_0 \to (i\omega)^{-1}v$. The same paper (equation (46)) gives also an expression for the displacement due to a transverse wave perturbation

$$u'_{transl} = \frac{\frac{\mu_2}{2R^2\omega}}{\frac{\mu_2}{2R^2\omega}\left(1 - ihR - \frac{1}{9}h^2R^2\right) + i\rho_{int}} v \qquad (13)$$

which is of the same order of magnitude as (12) at acoustic frequencies.

If we know the typical tissue vibrational velocity v, or equivalently the vibrational amplitude, we can then find the associated translational velocity u_{transl} of the lesion and through (1) the amount of signal detected by the Doppler. We notice that in the (unrealistic) limit of large ρ and ρ_{int} we can recover the naive result (5) from (12).

We can have a rough estimate for a typical value of the unperturbed vibrational velocity v of the mammalian tissue from Ref.[11]: here accelerometers were put on several points on the chest wall in order to detect the maximum amplitude of vibration during singing. For a non-professional singer the maximum displacement at sternum was $3 - 4\ \mu m$, which at voice frequency corresponds roughly to $v \sim 0.5$ cm/s; a precise measurement with accelerometers placed near the Doppler probe during the test has not been done yet. Nevertheless we notice that already $u_{transl} \sim v \sim 0.5$ cm/s may be enough to be detected by typical Doppler probes: for instance Esaote LA523 or LA435 are able to see such a signal. In Ref.[12] we can find the minimum velocity detected by the most common probes in the typical frequency range (5-10 MHz) used for a mammalian echo Doppler test: our signal is always above the threshold.

In conclusion the main contribution u_{transl} arising from the translational velocity seems to be enough to give the Doppler signal an important improvement, which could cast in the range of observability a signal which would not have been observed without "vocal humming".

IV. CONCLUSIONS

We described the frequency signal for an echo-Doppler test in a (mammalian) tissue vibrating at human voice frequency. We showed that the main contribution improving the signal/noise ratio is that of the translational motion of the different tissue components caused by the voice vibration and that voice has no other means of significantly influence the measurement (by e.g. changing the speed of propagation of ultrasounds in the underlying tissue). An important simplification is that the human voice frequency is low compared to the traveling period of the signal so that the vibrational velocity of the medium can be considered constant during an acquisition period: the systematic effect of having ultrasounds propagating in a moving medium is thus ruled out.

We calculated the amount of signal as a function of the vibrational amplitude and compared it with a typical range for an echo-Doppler device. These calculations seem to indicate that asking the patient to vocalize during the echo-Doppler test could be a good method to improve signal/noise ratio at no-cost. Of course an extensive clinical testing should be done in order to confirm these on-the-paper results.

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